# Pass-Through Impulse Response Functions (PT-IRFs)

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**Abstract**

Impulse response functions (IRFs) offer no insight regarding the channels through which a shock propagates through a dynamical system. I formulate the concept of a pass-through impulse response function (PT-IRF), which measures the passage of a structural shock through specific media in a given system. I demonstrate the applicability of the PT-IRF by performing inference on the effect of a monetary policy shock on unemployment through various channels of the monetary transmission mechanism using a simple structural vector autoregression.

**JEL Classifications:** C10; C22; C32; C50; E50

**Keywords:** Vector autoregressions; impulse response functions; monetary transmission

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# 1 Introduction

Often in macroeconomics we are interested in studying the dynamic effects of a particular shock on the economy, for which we default to impulse response functions (IRFs) as the tool of choice (Ramey, 2016). Given a dynamical process, an IRF captures the partial effect of a disturbance on the system over some specified time horizon. Empirical estimates of IRFs allow for the quantification of, and inference on, the effects of various economic shocks of interest on the macroeconomy – two common approaches to estimating IRFs include local projections (Jordà, 2005) and vector autoregressions (Stock and Watson, 2016). However, despite their ubiquity in the study of shock propagation, IRFs offer no insight into the nature of the channels contributing to the transmission of a shock through a system.

I formulate a new object, to which I henceforth refer as a pass-through impulse response function (PT-IRF), which allows for the isolation of specific transmission channels of a shock within a dynamical system. More specifically, given a dynamical system expressed in the form of a VAR, I propose an approach to estimating the effect of a structural shock $k$ on an endogenous variable $i$ through some other endogenous variable $k$, or a set of endogenous variables, as a medium. Conveniently, PT-IRFs can be estimated using the same information and procedures required to estimate IRFs in the context of VARs, which holds true for inference as well. The VAR case, general formulation, and estimation procedures for PT-IRFs are detailed in the given order in Section 2 of this paper.

Among other applications, PT-IRFs may be used to estimate, quantify, and conduct inference on various channels of the monetary transmission mechanism. Modern literature on the monetary mechanism has yet to reach an agreement on the roles of various transmission channels with respect to their contributions to the effects of monetary policy. For example, the literature on the credit channel, pioneered by Bernanke and Blinder (1992), remains inconclusive on the existence and nature of the bank lending channel. In Section 3 of this paper, I illustrate the potential of PT-IRFs in aiding such lack of consensus on the monetary transmission mechanism by estimating the separate and simultaneous pass-through of monetary policy shocks through bank lending and residential construction in a low-dimensional macroeconomic VAR. In the final section, I conclude the paper with a brief discussion of additional potential applications of PT-IRFs.

## 2 Methodology

In this section I define the concept of a PT-IRF starting with the simple case of a linear VAR(1), proceeding to the more general case of a linear VAR($p$), and finally generalizing to a stationary Markov process. I also describe how existing methods for estimating VAR IRFs can be used to produce point-estimates and confidence intervals for PT-IRFs.

### 2.1 Linear VAR(1)

Consider the following VAR(1) process:

$$Y_{t+1} = \alpha + AY_t + B\varepsilon_{t+1}\,, \tag{1}$$

where $Y_t = (y_{1t}, \ldots, y_{Nt})'$ is a vector of $N$ endogenous variables, $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Kt})'$ is a vector of $K$ structural shocks, $\alpha$ is an intercept vector, and $A \in \mathbb{R}^{N\times N}$ and $B \in \mathbb{R}^{N\times K}$ are the lag coefficient and impact matrices, respectively. Our goal is to interpret the given linear VAR(1) as a directed weighted graph through which shock impulses travel over time, use this alternative interpretation of a linear VAR(1) to reinterpret the familiar IRF, and finally define the PT-IRF within the given context.

Firstly, notice that the $ik$-th entry of $B$ represents the contemporaneous partial effect of the $k$-th structural shock on the $i$-th endogenous variable. Refer to Figure 1 for an illustration of the special case of a 3-dimensional VAR(1).

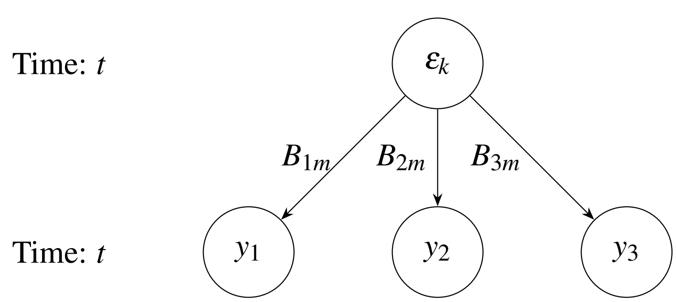


**Figure 1:** Contemporaneous effects of a structural shock $\varepsilon_k$ on a 3-dimensional VAR as a weighted directed graph. ***Note:*** Notice that for each $B_{ik}$, $i$ indexes the affected variable, while $k$ indexes the shock of origin.

Next, notice that $A_{ij}$ represents the one-period-ahead effect of the $j$-th variable on the $i$-th variable. If we think of $A$ as the adjacency matrix in the context of a directed weighted graph, where each endogenous variable at a given point in time is a vertex, then $A_{ij}$ my also be interpreted as the intensity of the travel path of a signal from variable $j$ at time $t$ to variable $i$ at time $t+1$. Once again, for an illustration of the above in the special case of a 3-dimensional VAR(1), refer to Figure 2.

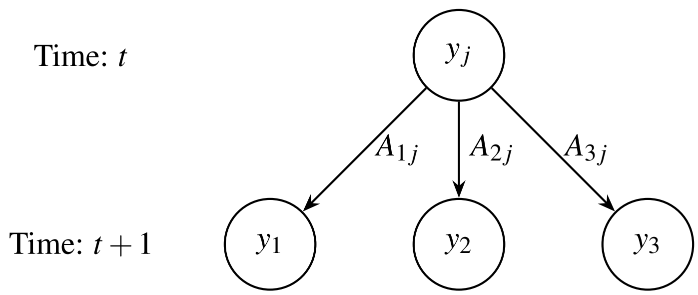


**Figure 2:** One-period-ahead effects of a change in the variable $y_j$ of a 3-dimensional VAR as a weighted directed graph. ***Note:*** Notice that for each $A_{ij}$, $i$ indexes the next period's destination variable, while $j$ indexes the variable of origin.

Lastly, we can simply put the above interpretations of $B$ and $A$ together to formulate a VAR(1) as a directed weighted graph, which allows us to trace the propagation of a shock through the system and assess its impact on some variable of interest at a future point in time. Each possible path of a given shock $\varepsilon_k$ has a corresponding weight equal to the product of the weights of each of its edges, determined by the impact and lag coefficient matrices. An IRF is simply the sum of the weights of all paths that ultimately reach a destination node corresponding to a variable of interest $y_i$ at a given horizon $h$. Refer to Figure 3 for an illustration of the one-period-ahead propagation of a shock through a 3-dimensional VAR(1).

A PT-IRF is the sum of weights associated with the subset of the above-mentioned paths that pass at least once through some medium of interest $y_j$ – if a given path never passes through $y_j$, then it is irrelevant in gauging the role of $y_j$ as a medium for a shock in the system. For example, the one-period-ahead pass-through response of $y_i$ with respect to $y_1$ as a medium to some shock $\varepsilon_k$ in the case illustrated by Figure 3 is equal to $A_{i1}B_{1k}$ – the weight of the only path that allows for the shock to pass through $y_1$ at least once before reaching its destination. If we were interested in the union of $y_1$ and $y_2$ as a medium for $\varepsilon_k$, then the PT-IR would be $A_{i1}B_{1k} + A_{i2}B_{2k}$ – the sum of the weights of the two paths that allow the shock to pass through either $y_1$ or $y_2$ at least once before reaching its destination. The same logic can be extended to $h$-period-ahead impulse responses, with $h$ being strictly greater than 1.

It can be shown that in the case of a VAR(1), the time-$t$ $h$-period-ahead impulse response (IR)

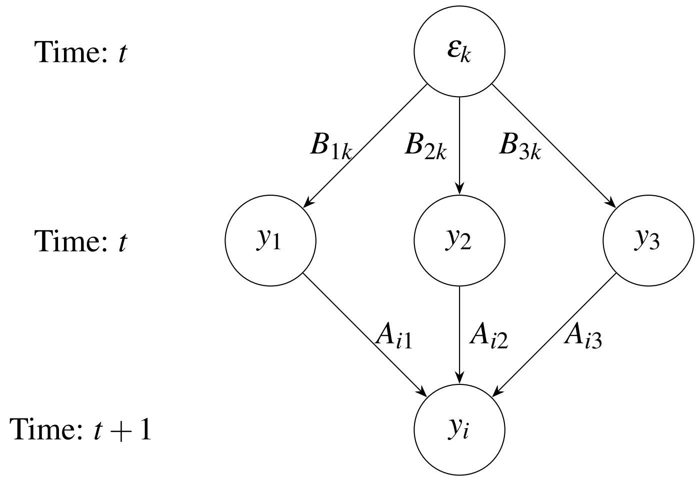


**Figure 3:** The propagation of an impulse originating at the $k$-th shock with the $i$-th variable as its destination, one period ahead. ***Note:*** The one-period-ahead impulse response of $y_i$ with respect to a unit shock to $\varepsilon_k$ equals the sum of the weights of all three paths leading to $y_{it+1}$: $A_{i1}B_{1k} + A_{i2}B_{2k} + A_{i3}B_{3k}$.

with respect to some vector of shocks $\overline{\varepsilon}$ may be expressed as

$$\text{IR}(t,h,\overline{\varepsilon}) = A^h B\overline{\varepsilon}\,. \tag{2}$$

It can also be shown that for $h > 0$, the corresponding pass-through impulse response (PT-IR) with pass-through/medium variable $y_j$ is algebraically equivalent to

$$\text{PT-IR}(t,h,j,\overline{\varepsilon}) \equiv \left(A^h - \widetilde{A}^h\right) B\overline{\varepsilon}\,, \tag{3}$$

where $\widetilde{A}$ is identical to $A$ across all but the $j$-th column, which is set equal to the zero vector. In the case that $h = 0$, the PT-IR always equals to zero due to the fact that contemporaneous pass-through of any given shock occurs purely through the impact matrix:

$$\text{PT-IR}(t,0,\overline{\varepsilon}) \equiv 0\,. \tag{4}$$

These final two equations completely define the PT-IRF in the context of a VAR(1).

## 2.2 Linear VAR($p$)

Consider the following VAR($p$) process:

$$Y_t = \alpha + A(L)Y_t + B\varepsilon_t \,, \tag{5}$$

where all familiar objects are defined as before, and $A(L)$ is a lag polynomial of the form

$$A(L) = \sum_{i=1}^{p} A_i L^i \,, \tag{6}$$

such that each $A_i$ is a lag coefficient matrix corresponding to $Y_{t-i}$. Suppose we aim to derive PT-IR$(t,h,j,\bar{\varepsilon})$ for this system. The goal is once again to sum the weights associated only with those paths that originate at the shock of interest, pass through $y_j$ at least once over the given horizon, and end at the response variable of interest $h$ periods ahead.

It can be shown that in the case of a linear VAR($p$), the following definition matches the above-described graph-theoretic intuition for PT-IRFs:

$$\text{PT-IR}(t,h,j,\bar{\varepsilon}) \equiv \text{IR}(t,h,\bar{\varepsilon}) - \widetilde{\text{IR}}(t,h,j,\bar{\varepsilon}) \,, \tag{7}$$

where $\widetilde{\text{IR}}(t,h,j,\bar{\varepsilon})$ is equivalent to $\text{IR}(\cdot)$ with matching parameters being applied to a modified version of the process described in Eq. (5) with the $i$-th lag coefficient matrix restricted to equaling

$$\widetilde{A}_i \equiv \begin{bmatrix} \vec{a}_1 & \dots & \vec{a}_{j-1} & \vec{0} & \vec{a}_{j+1} & \dots & \vec{a}_N \end{bmatrix} \,, \tag{8}$$

where $\vec{a}_m$ denotes the $m$-th column of $A_i$. In other words, $\widetilde{\text{IR}}(\cdot)$ captures the impulse response of a shock for a restricted version of the given linear VAR($p$) in which the Granger causality of the $j$-th endogenous variable is completely removed (Kilian and Lütkepohl, 2017) – all paths passing through the $j$-th variable are assigned a weight of zero. Therefore, $\text{PT-IR}(\cdot)$ sums the weights of only those paths that pass through the $j$-th variable, which can be interpreted as the impulse response of the system Granger-caused by the $j$-th endogenous variable.

## 2.3 General Formulation

Let $Y_t = (y_{1t}, \dots, y_{Nt})' \in \mathbb{R}^N$ and $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{Kt})' \in \mathbb{R}^K$ such that $Y_t$ is determined by the stationary Markov process

$$Y_t = G(\varepsilon_t, Y_{t-1}; \theta), \tag{9}$$

where $t \in \mathbb{N}^+$ denotes time, $G : \mathbb{R}^K \times \mathbb{R}^N \to \mathbb{R}^N$ is a mapping conditioned on a set of parameters $\theta$, and $\varepsilon_t$ is a vector of zero-mean i.i.d. shocks. We may define the $h$-step impulse response of the given system with respect to some shock vector $\overline{\varepsilon} \in \mathbb{R}^K$ as the following difference between two forecasts $\forall h \in \mathbb{N}$:

$$\mathrm{IR}(t,h,\overline{\varepsilon}) \equiv \mathbb{E}\left[Y_{t+h} \,|\, \varepsilon_t = \overline{\varepsilon}, Y_{t-1}, \theta\right] - \mathbb{E}\left[Y_{t+h} \,|\, \varepsilon_t = 0, Y_{t-1}, \theta\right], \tag{10}$$

where the conditional expectation operator $\mathbb{E}[\cdot\,|\,\cdot]$ represents the best mean squared error predictor. The pass-through impulse response of the system with respect to the same shock is once again defined as

$$\text{PT-IR}(t,h,j,\overline{\varepsilon}) \equiv \mathrm{IR}(t,h,\overline{\varepsilon}) - \widetilde{\mathrm{IR}}(t,h,j,\overline{\varepsilon}), \tag{11}$$

where $\widetilde{\mathrm{IR}}$ denotes an object similar to that in Eq. (10), but applied to a transformed version of the process expressed in Eq. (9). More specifically, we may define

$$\widetilde{\mathrm{IR}}(t,h,j,\overline{\varepsilon}) \equiv \mathbb{E}\left[\widetilde{Y}_{t+h} \,|\, \varepsilon_t = \overline{\varepsilon}, Y_{t-1}, \theta\right] - \mathbb{E}\left[\widetilde{Y}_{t+h} \,|\, \varepsilon_t = 0, Y_{t-1}, \theta\right], \tag{12}$$

where $\widetilde{Y}_t \equiv G(\widetilde{I}_j Y_{t-1}, \varepsilon_t; \theta)$, such that $\widetilde{I}_j$ is the identity matrix with the $j$-th diagonal entry set equal to zero. In other words, $\widetilde{Y}_t$ contains the variation in $Y_{t-1}$ attributable to the history of all but the $j$-th variable.

Notice that $\widetilde{I}_j$ removes the Granger causality of the $j$-th variable in the system, thus allowing for $\widetilde{\mathrm{IR}}(\cdot)$ to isolate the impulse response to a shock without accounting for its transmission through $y_{jt}$. Therefore, subtracting $\widetilde{\mathrm{IR}}(\cdot)$ from $\mathrm{IR}(\cdot)$ yields the impulse response associated with the transmission of a shock through $y_{jt}$. In conclusion, while the impulse response function (IRF), defined by Eq. (10), captures the expected net partial effect of a shock on the system, the pass-through impulse response function (PT-IRF), defined by Eq. (11), captures the expected partial effect of the pass-through of a shock through an endogenous variable $y_{jt}$.

### 2.4 Estimation

A PT-IRF may be estimated by first obtaining its corresponding IRF, and then using all relevant parameter estimates from this first step to generate the PT-IRF. Confidence intervals for a PT-IRF may be obtained in a similar manner – carry out the procedure necessary to generate IRF distributions (bootstrapping in the frequentist case, or sampling for a Bayesian approach) such that at each step of the bootstrap/sampler an accompanying PT-IRF is generated using the estimated parameters. All statistical properties of IRF estimators simply carry over to the estimation of PT-IRFs, since PT-IRFs essentially deterministic mappings of estimated parameters.

## 3 Applications

In this section, I illustrate the applicability of PT-IRFs to studying the channels of monetary transmission as an example. Stock and Watson (2001) estimate a simple recursively-identified three-dimensional VAR(4) to generate impulse response functions representing the effect of a one-time monetary policy shock on unemployment. Their model contains quarterly series on US inflation, unemployment, and the federal funds rate over the period of 1960:I-2000:IV. I introduce additional variables to their model, which I treat as media of interest for the transmission of monetary policy shocks through the system. Specifically, I use the additional variables to estimate separate and joint contributions of the bank lending and residential construction channels to the monetary transmission mechanism using PT-IRFs.

### 3.1 The Bank Lending Channel

Early literature on the bank lending channel (BLC), such as Bernanke and Gertler (1995), defines it as the effect of monetary policy on output through changes in the supply of bank loans. In other words, a monetary shock affects bank lending, which subsequently affects output. I add a commercial and industrial (C&I) loan growth rate series to the Stock and Watson (2001) VAR as the last variable in the recursive ordering, and use it as a pass-through medium in estimating the PT-IRF of unemployment to a monetary policy shock. The resulting IRF and PT-IRF are presented in Figure 4, which suggests that the bank lending acts as a substantial channel for monetary transmission. Furthermore, the shape of the PT-IRF matches the theory behind the BLC – a one-time contractionary policy shock causes a temporary rise in unemployment through a decrease in the growth of the supply of bank loans.

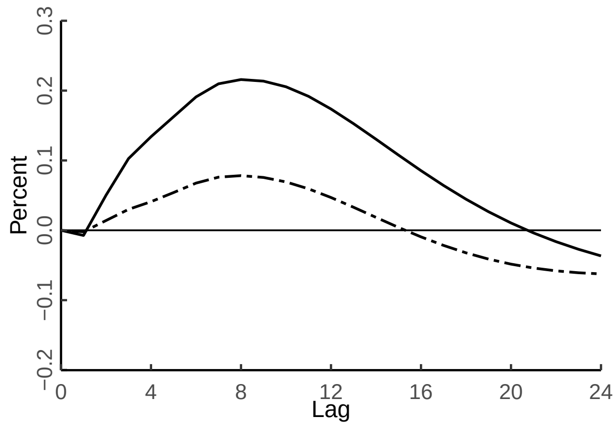


**Figure 4:** The IRF (solid line) and PT-IRF (dash-dotted line) of unemployment with respect to an interest rate shock, with bank lending as a pass-through medium.

## 3.2 The Residential Construction Channel

Another channel of monetary transmission is residential construction, as described by Mishkin (2007). To capture this channel using a VAR, I add a series representing the growth rate of total new privately-owned housing units authorized in permit-issuing places, which may be interpreted as a proxy for the growth of residential construction in the US. This most recent variable is ordered last in the resulting 5-variable VAR(4), which I use to estimate the IRF and PT-IRF of unemployment to a monetary shock, such that residential construction acts as a pass-through medium. Refer to Figure 5 for a presentation of the relevant impulse response plots, which suggest that residential construction plays a significant role in the transmission of monetary policy. Once again, the shape of the PT-IRF is intuitive – a contractionary policy shock causes a temporary increase in unemployment through a residential construction slowdown.

## 3.3 Combining Multiple Channels

Finally, I estimate monetary PT-IRFs with both bank lending and residential construction as pass-through media for monetary policy to demonstrate that PT-IRFs allow for more than one medium. Figure 6 presents the resulting PT-IRF along with its corresponding IRF. Notice that this PT-IRF matches that of the residential construction as an individual medium presented in Figure 5 – this

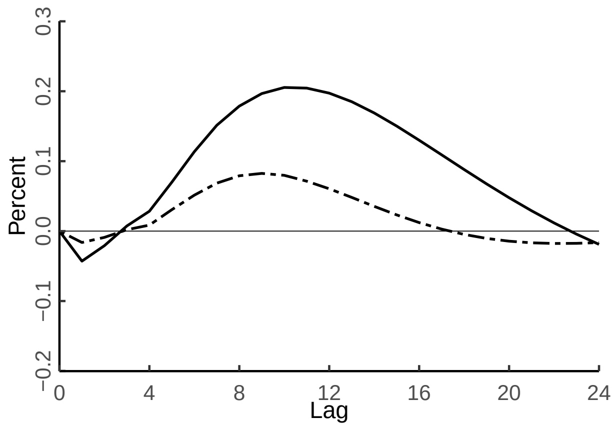


**Figure 5:** The IRF (solid line) and PT-IRF (dash-dotted line) of unemployment with respect to an interest rate shock, with residential construction as a pass-through medium.

implies a significant overlap between the path weights contributing to the bank lending and residential construction channels, as described by the graph-theoretic exposition in Section 2. In other words, there seems to be feedback between the two channels of monetary transmission.

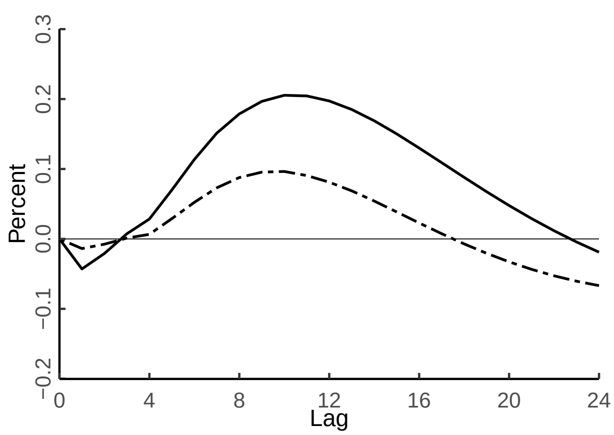


**Figure 6:** The IRF (solid line) and PT-IRF (dash-dotted line) of unemployment with respect to an interest rate shock, with bank lending and residential construction as pass-through media.

# 4 Concluding Remarks

PT-IRFs can be a useful tool for measuring the pass-through channels of various economic shocks. Although my applications in this paper involve linear VARs, PT-IRFs can easily be estimated for nonlinear VARs as well. Furthermore, the ability of PT-IRFs to accommodate multiple pass-through media allows for the estimation of multi-dimensional transmission channels – for example, in the case of the BLC, we could have simultaneously included multiple types of bank loan series as pass-through media. However, more work needs to be done to better understand how to interpret the PT-IRFs corresponding to unions of different pass-through channels – for example, the interpretation of the joint PT-IRF of the bank lending and residential construction channels is not completely clear.

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