Pass-Through Impulse Response Functions (PT-IRFs)

Giorgi Nikolaishvili*
University of Oregon

January 26, 2023

Abstract

Impulse response functions (IRFs) offer little insight regarding the channels by which a shock propagates through a dynamical system. I formulate the concept of a pass-through impulse response function (PT-IRF), which measures the passage of a structural shock through specific sets of variables in a given system. I demonstrate the applicability of PT-IRFs by measuring the strength of the effect of a monetary policy shock on unemployment through the credit channel of monetary transmission using a structural vector autoregression.

JEL Classifications: C10; C32; C50; E52

Keywords: Vector autoregressions; impulse response functions; monetary transmission

*I am grateful to Jeremy Piger, Bruce McGough, George Evans, David Evans, Jose Carreno, and participants of the University of Oregon Macro Group for many helpful comments and suggestions. All errors are my own. E-mail: gnikolai@uoregon.edu.

1 Introduction

Often in macroeconomics we are interested in studying the dynamic effects of a particular shock on the economy, for which we default to impulse response functions (IRFs) as the tool of choice (Ramey, 2016). Given a dynamical process, an IRF captures the effect of a disturbance on the system over some specified time horizon. Empirical estimates of IRFs allow for the quantification of, and inference on, the effects of various economic shocks of interest on the macroeconomy – two common approaches to estimating IRFs include local projections (Jordà, 2005) and vector autoregressions (Stock and Watson, 2016). However, despite their ubiquity in the study of shock propagation, IRFs offer little insight into the nature of the channels contributing to the transmission of a shock through a system.

I formulate a new object, to which I henceforth refer as a pass-through impulse response function (PT-IRF), which allows for the isolation of specific transmission channels of a shock within a dynamical system. More specifically, given a dynamical system expressed in the form of a VAR, I propose an approach to estimating the effect of a structural shock k on an endogenous variable i through some other endogenous variable j, or a set of endogenous variables, as a medium. Conveniently, PT-IRFs can be estimated using the same information and procedures required to estimate IRFs in the context of VARs, which holds true for inference as well. The VAR case, general formulation, and estimation procedures for PT-IRFs are detailed in Section 2 of this paper.

Among other applications, PT-IRFs may be used to estimate, quantify, and conduct inference on various channels of the monetary transmission mechanism. Modern literature on the monetary mechanism has yet to reach an agreement on the roles of various transmission channels with respect to their contributions to the effects of monetary policy. For example, the literature on the credit channel, pioneered by Bernanke and Blinder (1992), remains inconclusive on the existence and nature of the bank lending channel. In Section 3 of this paper, I illustrate the potential of PT-IRFs by estimating the pass-through of monetary policy shocks through bank lending in a low-dimensional macroeconomic VAR. In the final section, I conclude the paper with a brief discussion of additional potential applications of PT-IRFs.

2 Methodology

In this section I define the concept of a PT-IRF starting with the simple case of a linear VAR(1), proceeding to the more general case of a linear VAR(p), and finally generalizing to a stationary

Markov process. I also describe how existing methods for estimating VAR IRFs can be used to produce point-estimates and confidence intervals for PT-IRFs.

2.1 Linear VAR(1)

Consider the following VAR(1) process:

$$Y_{t+1} = \alpha + AY_t + B\varepsilon_{t+1}, \tag{1}$$

where $Y_t = (y_{1t}, \ldots, y_{Nt})'$ is a vector of N endogenous variables, $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Kt})'$ is a vector of K structural shocks, α is an intercept vector, and $A \in \mathbb{R}^{N\times N}$ and $B \in \mathbb{R}^{N\times K}$ are the lag coefficient and contemporaneous impact matrices, respectively. Our goal is to interpret the given linear VAR(1) as a directed weighted graph through which shock impulses travel over time, use this alternative interpretation of a linear VAR(1) to reinterpret the familiar IRF, and finally define the PT-IRF within the given context.

Firstly, notice that the ik-th entry of B represents the contemporaneous effect of the k-th structural shock on the i-th endogenous variable. Refer to Figure 1 for an illustration of the special case of a 3-dimensional VAR(1).

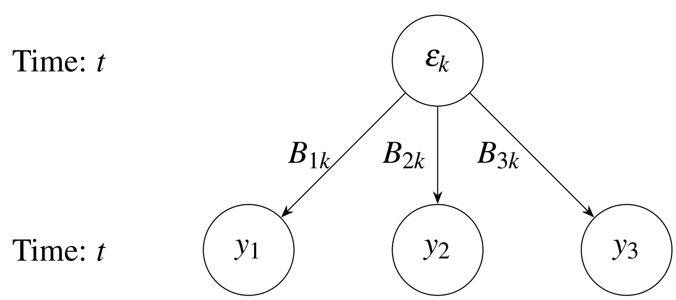

Figure 1: Contemporaneous effects of a structural shock ε_k on a 3-dimensional VAR as a weighted directed graph. ***Note:*** Notice that for each B_{ik}, i indexes the affected variable, while k indexes the shock of origin.

Next, notice that A_{ij} represents the one-period-ahead effect of the j-th variable on the i-th variable. If we think of A as the adjacency matrix in the context of a directed weighted graph, where each endogenous variable at a given point in time is a vertex, then A_{ij} may also be interpreted as the intensity of the travel path of a signal from variable j at time t to variable i at time $t+1$. Once again, for an illustration of the above in the special case of a 3-dimensional VAR(1), refer to Figure 2.

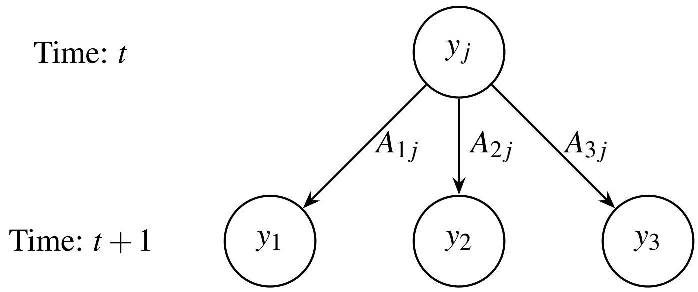

Figure 2: One-period-ahead effects of a change in the variable y_j of a 3-dimensional VAR as a weighted directed graph. ***Note:*** Notice that for each A_{ij}, i indexes the next period's destination variable, while j indexes the variable of origin.

Lastly, we can simply put the above interpretations of B and A together to formulate a VAR(1) as a directed weighted graph, which allows us to trace the propagation of a shock through the system and assess its impact on some variable of interest at a future point in time. Each possible path of a given shock ε_k has a corresponding weight equal to the product of the weights of each of its edges, determined by the contemporaneous impact and lag coefficient matrices. An IRF is simply the sum of the weights of all paths that ultimately reach a destination node corresponding to a variable of interest y_i at a given horizon h. Refer to Figure 3 for an illustration of the one-period-ahead propagation of a shock through a 3-dimensional VAR(1).

A PT-IRF is the sum of weights associated with the subset of the above-mentioned paths that pass at least once through some medium of interest y_j – if a given path never passes through y_j, then it is irrelevant in gauging the role of y_j as a medium for a shock in the system. For example, the one-period-ahead pass-through response of y_i with respect to y_1 as a medium to some shock ε_k in the case illustrated by Figure 3 is equal to $A_{i1}B_{1k}$ – the weight of the only path that allows for the shock to pass through y_1 at least once before reaching its destination. If we were interested in the union of y_1 and y_2 as a medium for ε_k, then the PT-IR would be $A_{i1}B_{1k} + A_{i2}B_{2k}$ – the sum of the weights of the two paths that allow the shock to pass through either y_1 or y_2 at least once before reaching its destination. The same logic can be extended to h-period-ahead impulse responses, with h being strictly greater than 1.

It can be shown that in the case of a VAR(1), the h-period-ahead impulse response (IR) with respect to some vector of shocks $\overline{\varepsilon}$ may be expressed as

$$\mathrm{IR}(h, \overline{\varepsilon}) = A^h B \overline{\varepsilon}. \tag{2}$$

It can also be shown that for $h > 0$, the corresponding pass-through impulse response (PT-IR) with

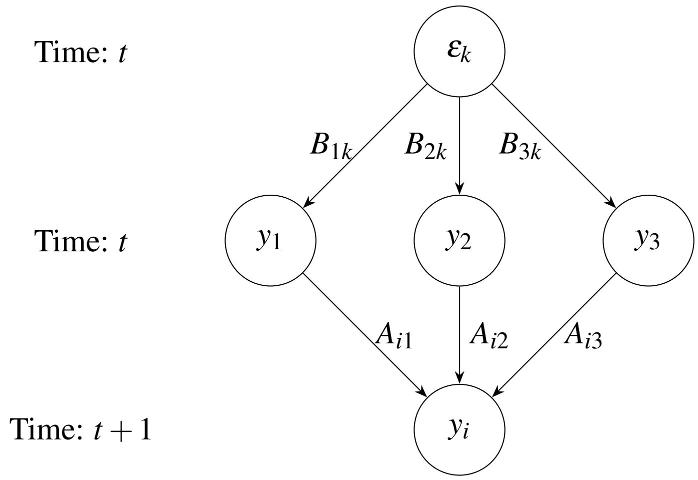

Figure 3: The propagation of an impulse originating at the k-th shock with the i-th variable as its destination, one period ahead. ***Note:*** The one-period-ahead impulse response of y_i with respect to a unit shock to ε_k equals the sum of the weights of all three paths leading to y_{it+1}: $A_{i1}B_{1k} + A_{i2}B_{2k} + A_{i3}B_{3k}$.

pass-through/medium variable y_j is algebraically equivalent to

$$\text{PT-IR}(h, j, \overline{\varepsilon}) \equiv \left(A^h - \widetilde{A}^h\right) B\,\overline{\varepsilon}\,, \tag{3}$$

where $\widetilde{A}$ is identical to A across all but the j-th column, which is set equal to the zero vector. In the case that $h = 0$, the PT-IR always equals to zero due to the fact that contemporaneous pass-through of any given shock occurs purely through the impact matrix:

$$\text{PT-IR}(0, j, \overline{\varepsilon}) \equiv 0\,. \tag{4}$$

The above equations completely define the PT-IRF in the context of a VAR(1).

2.2 Linear VAR(p)

Consider the following VAR(p) process:

$$Y_t = \alpha + A(L)Y_t + B\varepsilon_t\,, \tag{5}$$

where all familiar objects are defined as before, and $A(L)$ is a lag polynomial of the form

$$A(L) = \sum_{i=1}^{p} A_i L^i \,, \tag{6}$$

such that each A_i is a lag coefficient matrix corresponding to Y_{t-i}. Suppose we aim to derive PT-IR$(h, j, \overline{\varepsilon})$ for this system. The goal is once again to sum the weights associated only with those paths that originate at the shock of interest, pass through y_j at least once over the given horizon, and end at the response variable of interest h periods ahead.

Suppose we represent a linear VAR(p) in state-space form as a VAR(1) with companion matrix Φ and augmented contemporaneous impact matrix $\Gamma = \begin{bmatrix} B' & \mathbf{0} \end{bmatrix}'$. Then for $h \geq 0$ the corresponding PT-IR with pass-through/medium variable y_j may be expressed as

$$\text{PT-IR}(h, j, \overline{\varepsilon}) \equiv \left(\Phi^h - \widetilde{\Phi}^h\right) \Gamma \overline{\varepsilon} \,, \tag{7}$$

where $\widetilde{\Phi}$ is the companion matrix of a modified version of the process described in Eq. (5) with the i-th lag coefficient matrix restricted to equaling

$$\widetilde{A}_i \equiv \begin{bmatrix} \vec{a}_1 & \ldots & \vec{a}_{j-1} & \vec{0} & \vec{a}_{j+1} & \ldots & \vec{a}_N \end{bmatrix} , \tag{8}$$

where $\vec{a}_m$ denotes the m-th column of A_i. Notice that $\widetilde{\Phi}^h \Gamma \overline{\varepsilon}$ captures the impulse response of a shock for a restricted version of the given linear VAR(p) in which the Granger causality of the j-th endogenous variable is completely removed (Kilian and Lütkepohl, 2017) – all paths passing through the j-th variable are assigned a weight of zero. Therefore, PT-IR$(\cdot)$ sums the weights of only those paths that pass through the j-th variable, which can be interpreted as the impulse response of the system attributable to the Granger-causality of the j-th endogenous variable.

2.3 General Formulation

Let $Y_t = (y_{1t}, \ldots, y_{Nt})' \in \mathbb{R}^N$ and $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Kt})' \in \mathbb{R}^K$ such that Y_t is determined by the stationary Markov process

$$Y_t = G(\varepsilon_t, Y_{t-1}; \theta) \,, \tag{9}$$

where $t \in \mathbb{N}^+$ denotes time, $G : \mathbb{R}^K \times \mathbb{R}^N \to \mathbb{R}^N$ is a mapping conditioned on a set of parameters θ, and ε_t is a vector of zero-mean i.i.d. shocks. We may define the h-step impulse response of the given system with respect to some shock vector $\overline{\varepsilon} \in \mathbb{R}^K$ as the following difference between two forecasts $\forall h \in \mathbb{N}$:

$$\mathrm{IR}(t,h,\overline{\varepsilon}) \equiv \mathbb{E}\left[Y_{t+h} \,|\, \varepsilon_t = \overline{\varepsilon},\, Y_{t-1},\, \theta\right] - \mathbb{E}\left[Y_{t+h} \,|\, \varepsilon_t = 0,\, Y_{t-1},\, \theta\right], \tag{10}$$

where the conditional expectation operator $\mathbb{E}[\cdot\,|\,\cdot]$ represents the best mean squared error predictor. The pass-through impulse response of the system with respect to the same shock is once again defined as

$$\text{PT-IR}(t,h,j,\overline{\varepsilon}) \equiv \mathrm{IR}(t,h,\overline{\varepsilon}) - \widetilde{\mathrm{IR}}(t,h,j,\overline{\varepsilon}), \tag{11}$$

where $\widetilde{\mathrm{IR}}$ denotes an object similar to that in Eq. (10), but applied to a transformed version of the process expressed in Eq. (9) in which the Granger causality of the j-th variable in the system is removed. More specifically, we may define

$$\widetilde{\mathrm{IR}}(t,h,j,\overline{\varepsilon}) \equiv \mathbb{E}\left[\widetilde{Y}_{t+h} \,|\, \varepsilon_t = \overline{\varepsilon},\, Y_{t-1},\, \theta\right] - \mathbb{E}\left[\widetilde{Y}_{t+h} \,|\, \varepsilon_t = 0,\, Y_{t-1},\, \theta\right], \tag{12}$$

where $\widetilde{Y}_t \equiv G(\varepsilon_t, \widetilde{I}_j Y_{t-1}; \theta)$, such that $\widetilde{I}_j$ is the identity matrix with the j-th diagonal entry set equal to zero. In other words, $\widetilde{Y}_t$ is generated by the same process as Y_t, but with the influence of the lags of the j-th variable removed from the data generating process (DGP).

Notice that $\widetilde{I}_j$ removes the Granger causality of the j-th variable in the system, thus allowing for $\widetilde{\mathrm{IR}}(\cdot)$ to isolate the impulse response to a shock without accounting for its transmission through y_{jt}. Therefore, subtracting $\widetilde{\mathrm{IR}}(\cdot)$ from $\mathrm{IR}(\cdot)$ yields the impulse response associated with the transmission of a shock through y_{jt}.

2.4 Estimation

A PT-IRF may be estimated by first obtaining its corresponding IRF, and then using all relevant parameter estimates from this first step to generate the PT-IRF. Confidence intervals for a PT-IRF may be obtained in a similar manner – carry out the procedure necessary to generate IRF distributions (bootstrapping in the frequentist case, or sampling for a Bayesian approach) such that at each step of the bootstrap/sampler an accompanying PT-IRF is generated using the estimated parameters. All statistical properties of IRF estimators simply carry over to the estimation of PT-IRFs, since PT-IRFs are deterministic mappings of estimated parameters.

3 Application

In this section, I apply PT-IRFs to study monetary transmission. Stock and Watson (2001) estimate a simple recursively-identified three-dimensional VAR(4) to generate impulse response functions representing the effect of a one-time monetary policy shock on unemployment. Their model contains quarterly series on US inflation, unemployment, and the federal funds rate over the period of 1960:I-2000:IV. I introduce an additional variable to their model, which I treat as medium of interest for the transmission of monetary policy shocks through the system. I estimate a PT-IRF to measure the strength of the bank lending channel (BLC) in the monetary transmission mechanism.

Early literature on the BLC, such as Bernanke and Gertler (1995), define it as the effect of monetary policy on output through changes in the supply of bank loans. In other words, a monetary shock affects bank lending, which subsequently affects output. I add a commercial and industrial (C&I) loan growth rate series to the Stock and Watson (2001) VAR as the last variable in the recursive ordering, and use it as a pass-through medium in estimating the PT-IRF of unemployment to a monetary policy shock. The resulting IRF and PT-IRF are presented in Figure 4, which suggests that bank lending acts as a substantial channel for monetary transmission. Furthermore, the shape of the PT-IRF matches the theory behind the BLC – a one-time contractionary policy shock is associated with a temporary rise in unemployment through a decrease in the growth of the supply of bank loans.

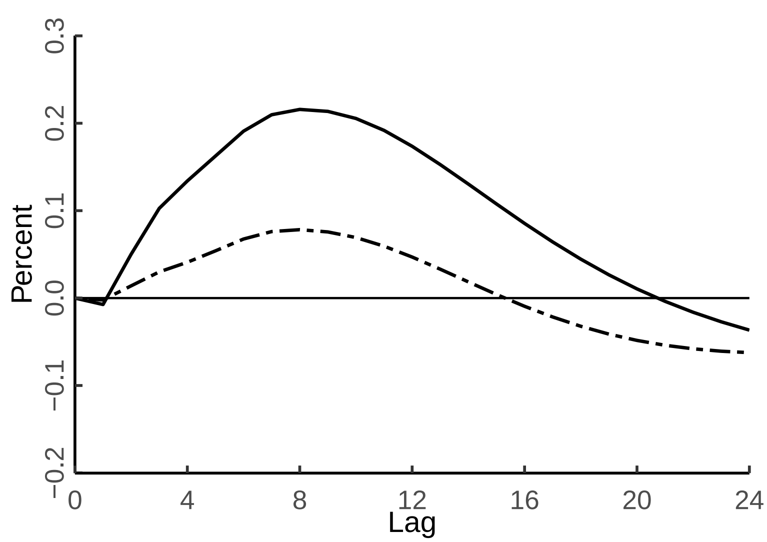

Figure 4: The IRF (solid line) and PT-IRF (dash-dotted line) of unemployment with respect to an interest rate shock, with bank lending as a pass-through medium.

4 Concluding Remarks

PT-IRFs can be a useful tool for measuring the pass-through channels of various economic shocks. Although my application in this paper involves a linear VAR, PT-IRFs can easily be estimated for nonlinear VARs as well. Furthermore, the ability of PT-IRFs to accommodate multiple pass-through media allows for the estimation of multi-dimensional transmission channels – for example, in the case of the BLC, we could have simultaneously included multiple bank loan type series as pass-through media.

References

Bernanke, B. S. and Blinder, A. S. (1992). The federal funds rate and the channels of monetary transmission. *The American Economic Review*, 82(4):901–921.

Bernanke, B. S. and Gertler, M. (1995). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives*, 9(4):27–48.

Jordà, O. (2005). Estimation and inference of impulse responses by local projections. *American Economic Review*, 95(1):161–182.

Kilian, L. and Lütkepohl, H. (2017). *Vector Autoregressive Models*, page 19–74. Themes in Modern Econometrics. Cambridge University Press.

Ramey, V. (2016). Chapter 2 - macroeconomic shocks and their propagation. In Taylor, J. B. and Uhlig, H., editors, *Handbook of Macroeconomics*, volume 2, pages 71–162. Elsevier.

Stock, J. and Watson, M. (2016). Chapter 8 - dynamic factor models, factor-augmented vector autoregressions, and structural vector autoregressions in macroeconomics. In Taylor, J. B. and Uhlig, H., editors, *Handbook of Macroeconomics*, volume 2, pages 415–525. Elsevier.

Stock, J. H. and Watson, M. W. (2001). Vector autoregressions. *Journal of Economic Perspectives*, 15(4):101–115.